EXERCISE BAT5: COMPOSITE COLUMN - SOLUTION

Question 1.1

Resistance under normal force

Plate buckling (section class):

Unnecessary verification if coverage equal to or greater than 40 mm, or b/6:

HEA 140: $c_z \geq (270-140)/2 = 65$ mm > 40 mm **OK**

Section characteristics:

$A_a = 3140\ \text{mm}^2$

$A_s = 8 \cdot 254.5 = 2040\ \text{mm}^2$

$A_c = A_{tot} - Aa - As = 67700\ \text{mm}^2$

$I_{az} = 3.89 \cdot 10^6\ \text{mm}^4$

$I_{sz} = 2 \cdot 3 \cdot \pi \cdot 92 \cdot 96^2 = 14.1 \cdot 106\ \text{mm}^4$ (self-inertia neglected)

$$I_{cz} = \frac{b \cdot h^3}{12} - I_{az} - I_{sz} = \frac{270^4}{12} - 3.89 \cdot 10^6 - 14.1 \cdot 10^6 = 425 \cdot 106\ \text{mm}^4$$

Facultative: $n_{pl} = \frac{f_y \cdot \gamma_c}{\gamma_a \cdot 0{,}85 f_{ck}} = \frac{235 \cdot 1{,}5}{1{,}05 \cdot 0{,}85 \cdot 40} = 9{,}87$

$$n_{pls} = \frac{f_y \cdot \gamma_s}{f_{ysk} \cdot \gamma_a} = \frac{235 \cdot 1{,}15}{500 \cdot 1{,}05} = 0{,}515$$

Checking the reinforcement ratio

$\rho = 0.6\% \leq A_s/A_c = 3\ \% \leq \rho = 6\ \%$ → **OK**

Lower limit: ρ = 0.6 % according to SIA 262, Art. 5.5.4.2.

Upper limit: ρ = 6 % in general and in particular for concrete-filled hollow sections (according to SIA 264, Table 7)

ρ = 8 % is possible for section embedded in concrete (according to SIA 262, Art. 5.5.4.5).

Verification of the validity of the verification method for composite columns

$$0.2 < \delta = \frac{N_{pl,Rd,HEA140}}{N_{pl,Rd,mixte}} < 0.9$$

Formula from SIA 264 § 5.3.2.4 :

$$N_{pl,Rd,mixte} = A_a \frac{f_y}{\gamma_a} + 0.85 \cdot A_c \frac{f_{ck}}{\gamma_c} + A_s \frac{f_{sk}}{\gamma_s} = 3.14 \cdot 10^3 \cdot \frac{235}{1.05} + 0.85 \cdot 67.7 \cdot 10^3 \cdot \frac{40}{1.5} + 2.04 \cdot 10^3 \cdot \frac{500}{1.15}$$
$$= 3.12 \cdot 10^3\ \text{kN}$$

Note: $N_{pl,Rd,mixte}$ can also be determined by means of the equivalence coefficients (formula from course):

$$N_{pl,Rd,mixte} = \frac{f_y}{\gamma_a}\left(A_a + \frac{A_c}{n_{pl}} + \frac{A_s}{n_{pls}}\right) = \frac{235}{1.05}\left(3.14 \cdot 10^3 + \frac{67.7 \cdot 10^3}{9.9} + \frac{2.04 \cdot 10^3}{0.51}\right) = 3.13 \cdot 10^3 kN$$

$\delta = 0{,}23$ (SZS C5 p.24: $N_{pl,Rd,HEA\ 140} = 703$ kN) → **OK**

Elastic buckling critical load (with $EI_{eff,\lambda}$)

$l_{K,z} = l = 5$ m

$E_a = E_s = 210 \cdot 103$ N/mm^2

$E_{cm} = 40 \cdot 103$ N/mm^2

As a general rule, the following conservative relationship $E_c = E_{cm} / 2.5$ can be taken as a simplification to account for the long-term effects (creep and shrinkage) in compressed elements (TGC 10 p.267). By making more precise calculations (SIA 264, Art. 5.3.2.9), a more favourable module can be found ; i twill be used in the calculations below.

$Ec = E_{cm} / 2 = 20 \cdot 103$ N/mm2 (with the following assumptions: $\varphi = 2$ *and* $N_{G,Ed} / N_{Ed} = 0.5$)

$(EI)_{Eff\lambda} = E_\alpha I_{az} + E_s I_{sz} + 0.6\ E_c I_{Cz}$

$= 2.1 \cdot 10^5 \cdot 3.89 \cdot 10^6 + 2.1 \cdot 10^5 \cdot 14.1 \cdot 10^6 + 0.6 \cdot 2 \cdot 10^4 \cdot 425 \cdot 10^6 = 8.88 \cdot 10^{12}$ Nmm^2

$N_{crz,\ mixte} = N_{crz} = \pi^2 \cdot (EI)_{eff,\lambda} / l_{K,z}^2 = 3.51 \cdot 10^3$ kN

Structural Safety Verification

Buckling check:

Checking the slenderness coefficient to determine if the simplified method of design under centered compression is valid:

The characteristic value of the plastic resistance to normal stress is:

$N_{pl,k,mixte} = 3140 \cdot 235 + 67700 \cdot 0.85 \cdot 40 + 2040 \cdot 500 = 4.06 \cdot 10^3$ kN

$$\bar{\lambda}_{Kz} = \sqrt{\frac{N_{pl,k}}{N_{crz}}} = \sqrt{\frac{4.06 \cdot 10^3}{3.51 \cdot 10^3}} = 1.08 \leq 2 \Rightarrow valable$$

2nd order effect? Eccentricity of the load, e ? Not specified, so we can admit $e\ \cong 0$ ($and\ \leq 0.1h$ thus only a small increase if the 2nd order would be considered).

As a result, the ratio $\frac{N_{Ed}}{N_{crz,eff}}$ is irrelevant, the buckling curves already include an imperfection (out of straightness, positioning of the load). So no 2nd order, direct use of buckling curves possible.

Curve **c** according to SIA 264 Table 7. From SIA 263, fig. 7:

$\chi_{Kz} = 0.49$

$\boldsymbol{N_{Kz,Rd,mixte}} = \chi_{Kz} \cdot N_{pl,Rd} = 0.49 \cdot 3.12 \cdot 10^3 = 1.53 \cdot 10^3$ kN

$\boldsymbol{N_{Kz,Rd,mixte}} = 1.53 \cdot 10^3\ \boldsymbol{kN} \geq N_{ed} = 1.5 \cdot 10^3$ kN → **OK**

Checking for the introduction of the normal force at the extremity:

The proportion of the force which must be transferred from the steel to the concrete and the reinforcement is obtained in relation to the proportions $N_{i,Ed} = N_{Ed} \frac{N_{pl,i}/\gamma_{Mi}}{N_{pl,Rd}}$. The shear force to be transferred is therefore equal to:

$$V_{L,Ed} = N_{c,Ed} + N_{s,Ed} = N_{Ed}\left[1 - \frac{N_{pl,a}/\gamma_a}{N_{pl,Rd}}\right] = 1500 \cdot 10^3 \left[1 - \frac{3140 \cdot \frac{235}{1.05}}{3130 \cdot 10^3}\right] = 1163.2\ kN$$

Introducing length: $L_{intro} = 2d = 2 \cdot 270 = 540$ mm

The perimeter of an HEA 140 can be found in the SZS C5: $U_m = 0.795$ m^2/m

Introducing area: $A = U_m \cdot L_{intro} = 0.794 \cdot 10^3 . 540 = 428'800$ mm^2

The shear strength for a profile fully embedded is as follows: $\tau_{Rd} = 0.30$ N/mm^2

Verification:

$$\tau_{Ed} = \frac{V_{L,Ed}}{A} = \frac{1163200}{428800} = 2{,}71 \leq ?\ \tau_{Rd} = 0{,}30\ N/mm^2$$ **KO!**

Solutions: either put studs, or and this is probably the case here, have an adequate detailing for introducing the force, for ex. using an head plate:

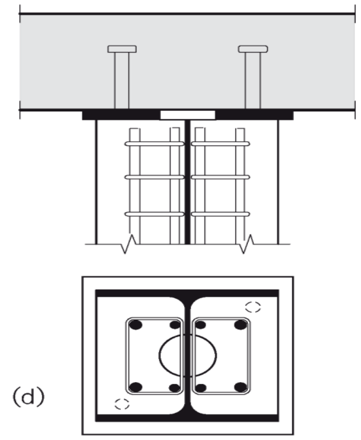

***Figure 1** – Detailing of the connection between slab and composite column*

Question 1.2

The compression-bending design method presented in § 6.5.3 of TGC 10 is applicable, see question 1.1. Only the key results are presented here. For more information, see TGC 10, § 6.5.3 and numerical example 6.7.

Calculation of the M-N interaction curve of the cross-section

The curve will be composed of the three points A, B and C. The composite cross-section is bend along the weak axis of the profile.

Coordinates of point A:

$M_{point\ A} = 0$

$N_{point\ A} = N_{pl,Rd} = 3.12 \cdot 10^3$ kN (see question 1.1)

Point B Coordinates:

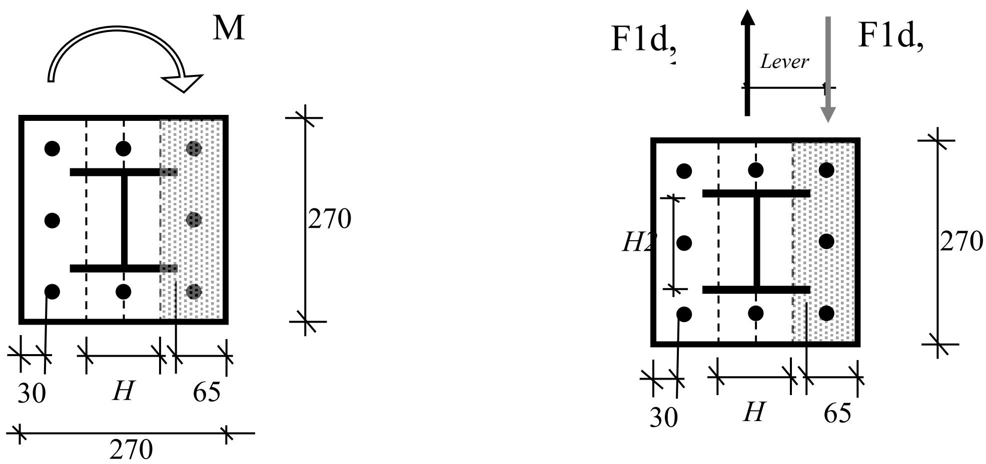

***Figure 2** – Dimensions for calculating the coordinates of the different points of the interaction curve*

We start by determining the position of the neutral axis:

Assumptions: cracked concrete neglected, reinforcing bars considered as concrete (because the + and – forces compensate each other approximately), neglected profile root fillets and neutral axis assumed to be within the steel profile section.

$$F_{1d,béton} = 0.85\frac{f_{ck}}{\gamma_c}\left[270\cdot 65 + (270-2t_f)\left(\frac{b}{2}-\frac{h_0}{2}\right)\right] = 7.99\cdot 10^5 - 2.87\cdot 10^3\cdot h_0$$

$$F_{1d,acier} = \frac{f_y}{\gamma_a}\left(h_2\cdot t_w + 2\cdot h_0\cdot t_f\right) = 1.43\cdot 10^5 + 3.81\cdot 10^3\cdot h_0$$

$F_{1d,\ concrete} = F_{1d,\ steel} = F_{1d}$

We find h_0 = 98mm (h_0 = 98mm < b = 140mm so the neutral axis is within the steel profile section) and F_{1d} = 517 kN

Compression table x = $(b_{concrete}-h_0)/2$

Note: The 4 extremities of the steel section outside the area bounded by h_0 (see figure 1) are not written in the equation $F_{1d,\ concrete} = F_{1d}$, steel because their contributions appear on the right and also on the left side of the equation, i.e. $2t_f(b/2-h_0/2)\ f_y/\gamma_a$

The moment taken up by the concrete part and the central part of the steel profile is: $M_{1d} = F_{1d}\cdot L_{lever} = F_{1d}\cdot (h_0/2+x/2) = F_{1d}\cdot (b_{concrete}/4+h_0/4)$ = 517 · 103 · (270/4 + 98/4) = 47.8 kNm. The lever arm is equivalent to the distance between the centres of gravity of the areas subject to $F_{1d\ steel}$ and $F_{1d\ concrete.}$

The moment from the four equation of the flanges of the steel profile is:

$$M_{2d} = 2\cdot\frac{f_y}{\gamma_a}\left(\frac{b}{2}-\frac{h_0}{2}\right)\cdot t_f\cdot\left(\frac{b}{2}-\frac{h_0}{2}+h_0\right) = 9.43kNm$$

The moment from the rebar (2 middle bars neglected, because their contribution to the moment is negligible) is worth:

$$M_{3d} = \frac{f_{sk}}{\gamma_s}\cdot 3\cdot\pi\cdot r^2\cdot\left(270-2\cdot 30-2r\right) = 63.7kNm$$

Hence: $M_{point\ B} = M_{pl,Rd}$ = M1d +M2d+M3d = 121 kNm

$N_{point\ B} = 0$

Note: Different assumptions on the more or less exact consideration of the reinforcing bars for the calculation of the neutral axis and the lever arm of the concrete block give values of $M_{pl,Rd}$ between 118 kNm and 123 kNm, thus all can be considered valid.

Coordinates of point C:

$$N_{point C} = N_{pm,Rd} = \frac{2f_y}{\gamma_a}\left(h_2\cdot t_w + 2h_0\cdot t_f\right) + 0.85\frac{f_{ck}}{\gamma_c}\left(270\cdot h_0 - h_2\cdot t_w - 2h_0\cdot t_f\right) = 1.58\cdot 10^3 kN$$

$M_{point\ C} = M_{point\ B} = M_{pl,Rd}$

Note: t_f, h_2, b, t_w are dimensions relative to HEA 140, see SZS C5.

Verification of the M-N interaction

Elastic buckling critical load (with $EI_{eff,d}$)

We have to recalculate with a reduced effective stiffness due to bending

$(EI)_{eff,d} = 0.9(E_\alpha I_{az} + E_s I_{sz} + 0.5\ E_c I_{Cz})$

= 0.9 (2.1·105 · 3.89 · 106 + 2.1·105 · 14.1 · 106 + 0.5 · 2·104 · 425 · 106) = 7.23 · 10^{12} Nmm2

(which is significantly lower than $(EI)_{eff,\lambda}$)

$N_{crz,\,eff} = \pi^2 \cdot (EI)_{eff,d} / l_{K,z}^2 = 2.852 \cdot 10^3$ kN (which is significantly lower than the previous value N_{crz})

Design internal forces:

$M_{z,Ed,\,max} = Q_{d,shock} \cdot L / 4 = 60 \cdot 5 / 4 = 75$ kNm

$N_{Ed} = 762$ kN

With N_{Ed}, we find by means of the interaction curve that:

$N_{Ed} < N_{pm,Rd}$ so in zone BC: $M_{z,pl,N,Rd} = M_{z,pl,Rd} = 121$ kNm

$$\frac{N_{Ed}}{N_{crz,eff}} = \frac{762}{28520} = 0.267 > 0.1$$

So we have to check if we should consider buckling with the 2nd direct order, and since the moment is not linear, $\omega_z = 1.0$. In EN1994-1-1, we have the factor β instead ω, which is obviously also 1.0 for our moment diagram.

$$k = \frac{\omega_z}{1 - N_{Ed}/N_{crz,eff}} = \frac{1}{1 - 0.267} = 1.364 > 1.0$$

The 1st order moment must be amplified to account for buckling. In our case, it is the section at mid-height that is the most stressed, in the 1st as well as the 2nd order.

The example of TGC 10 is not consistent with the method of EN 1994-1-1 and SIA264:2014. If we follow TGC 10, one will determine:

- The amplification k from $N_{crz,\,mixte}$ and not $N_{crz,\,mixte\,d}$, so obtain $k = 1.28$
- The moment at mid-height as $M_{Ed,max,II} = k \cdot M_{z,Ed,max} = 1.28 \cdot 75 = 96$ kNm < 121 kNm, and one concludes that the verification is easily satisfied. But this doesn't take into account the e_I imperfection, so doing so is incomplete and non-conservative.

Determination of the term *of* $k N_{Ed} \cdot e_I$ and verification ($M_{Ed,max,I}$ section at mid-height)

According to SIA 264 and EN1994-1-1:

$$M_{Ed,max,II} = k\left(M_{Ed,max,I} + N_{Ed} \cdot e_1\right) = 1.364 \cdot (75 + 762 \cdot 0.033) = 136.6\ kNm$$

With, according to SIA 264, tab. 7: $e_1 = L/150 = 5000/150 = 33\ mm$

With the value of N_{Ed} and the interaction curve, we are in the zone: $\mu_d = 1.0$

$$\frac{M_{Ed,max,II}}{M_{pl,N,Rd}} = \frac{M_{Ed,max,II}}{\mu_d \cdot M_{pl,Rd}} = \frac{136.6}{121} = 1{,}13 > 0.9$$ → **KO!**

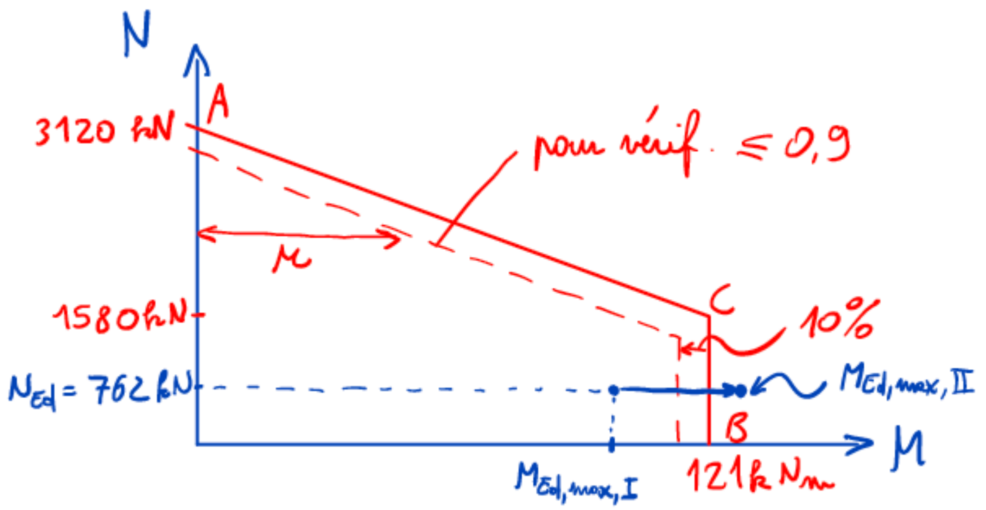

***Figure 3** – Interaction (plain) and verification (dashed) curves (-10%)*

One probably has to use an HEA 160 instead. This increases the compressive strength of the steel profile by 868/703 = 23% and the buckling strength by 324/218 = 48%. Thus this should be sufficient.

Shear Strength Verification (HEA 140)

With the conservative assumption that the steel profile carries it all

$V_{Ed} = Q_{shock,Ed}/2 = 30$ kN < 50% $V_{pl,a,Rd} = 0.5 \cdot 131 = 65.5$ kN → OK, no M-V interaction (for any profile ≥ HEA 140)

Note: the value of k depends not on one but on all moment diagrams. When the shape of the diagram of moments M_{Ed} differs from that of the diagram corresponding to the imperfection e_1 (which is always parabolic) and the result is a value of β_2 or $\omega_2 < 1.0$ for M_{Ed}, then this favorable effect can be taken into account. The factors k_1 and k_2 are determined separately and the second-order moment is then calculated as:

$$M_{Ed,max,II} = k_2 \cdot M_{Ed,max,I} + k_1 \cdot N_{Ed} \cdot e_1$$

Verification of the introduction of the transverse load (HEA 140)

This time it is a question of checking the shear caused by the transverse force acting at mid-height of the column.

Assumption: For simplicity, it is assumed that the shear force must be able to be transferred entirely from the steel to the concrete, or vice versa.

$V_{Ed} = Q_{shock,Ed}/2 = 30$ kN to be introduced on $L_{intro} = 2d = 2*270 = 540$ mm

Introducing area: $A = U_m \cdot L_{intro} = 0.795 \cdot 10^3 \cdot 540 = 428'800$ mm2

Verification:

$$\tau_{Ed} = \frac{V_{Ed}}{A} = \frac{30000}{428800} = 0{,}07 \leq \tau_{Rd} = 0{,}30\ N/mm^2$$ **OK**

Notes:

- The unsatisfied checks should be redone with HEA 160 (for those that have been satisfied, such as the introduction of the transverse load, it is not necessary, they will be more favourable),
- Sizing of the studs at the ends, or designing an appropriate introduction detailing such as an head plate (Figure 1),
- There are other load cases to check, with other N-M combinations, which could end up to be more unfavorable.